

# Internal and External Reliability in the Paraná Scientific Gravity Network

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## Resumo

No presente trabalho são apresentadas as concepções, estratégias e atividades experimentais associadas para nova metodologia de implantação de redes gravimétricas de alta precisão no Brasil. O levantamento das observações foi efetuado com quatro gravímetros, três do tipo LaCoste&Romberg e um gravímetro digital Scintrex. O principal objetivo deste trabalho é apresentar e aplicar o critério para confiabilidade para redes geodésicas. Conclui-se o trabalho fazendo uma análise de qual das soluções melhor concebe a rede gravimétrica, quanto à precisão e confiabilidade.

*Palavras-chave:* Rede gravimétrica, Critério para precisão, Critério para confiabilidade.

## Abstract

This work presents the conceptions, strategies and experimental activities associated with a new methodology of establishing high precision gravity network in Brazil. The survey was accomplished with four spring gravimeters, three LaCoste & Romberg and a digital Scintrex gravimeter. The principal objective of this work is to present and to apply the criteria for geodesic network reliability. The work makes an analysis of which solutions for the network is better, taking into account the precision and reliability.

*Keywords:* Gravity Network Criteria for precision, Criteria for reliability.

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## **Introduction**

Due to the necessity to generate a precise gravity network, the creation of a scientific gravity network in the State of Paraná was proposed. The control of a scientific gravity network shows that the aim is the constitution of a reference gravity network with a precision higher than all the other Brazilian relative networks. For instance, the Brazilian gravimetric network of first order, in general has a precision between 50 and 100 microgal. So, the aim is to generate a network, which can answer the scientific proposals with a precision better than 30 microgal. Due to this, different pondering and adjustment strategies are being used. Hence the control of a scientific network.

The data survey work was developed by the Federal University of Paraná, throughout three weeks, scouring the whole State, totaling more than 12000 kilometers. In this work the following gravimeters were used; three La Coste & Romberg G-114 and G-143 models borrowed from the Brazilian Geographic and Statistic Institute (IBGE), G-372 of the Federal University of Paraná and a Scintrex digital gravimeter, model CG-3, borrowed from the Laboratory of Geophysics Research of the Federal University of Paraná. As a result, we obtained a consistent volume of observation, which made it possible to compare the results of the different gravimeters, as well as to determine the average values for all network points, starting from an adjusting process.

## **Network characteristics**

The points used during the implantation of the scientific gravity network were the High very well spread Precision GPS network stations of Paraná, as shown in Figure 1. All the marks of this network are implanted in protected areas, that diminishes the risk of losing the point. The only exception was the station located in Curitiba at the state owned electric company called COPEL, because in this city we have the absolute gravity station implanted by UFPR.

Due the configuration of the marks, in most of the cases, it was not possible to carry out the readings on the upper part, where the forced centering system is placed which was objective of this work. In these cases, the observations were carried out on the triangular low foot, pillar prop, imposing the need to determine the pillar height to proceed to the necessary reductions as shown at Figure 2.

Only in the observations carried out at Ponta Grossa, in the State University of Maringa and at the Farming Municipal Center at Larajeiras do Sul, was it possible to occupy the upper part of the mark, as seen at Figure 3.

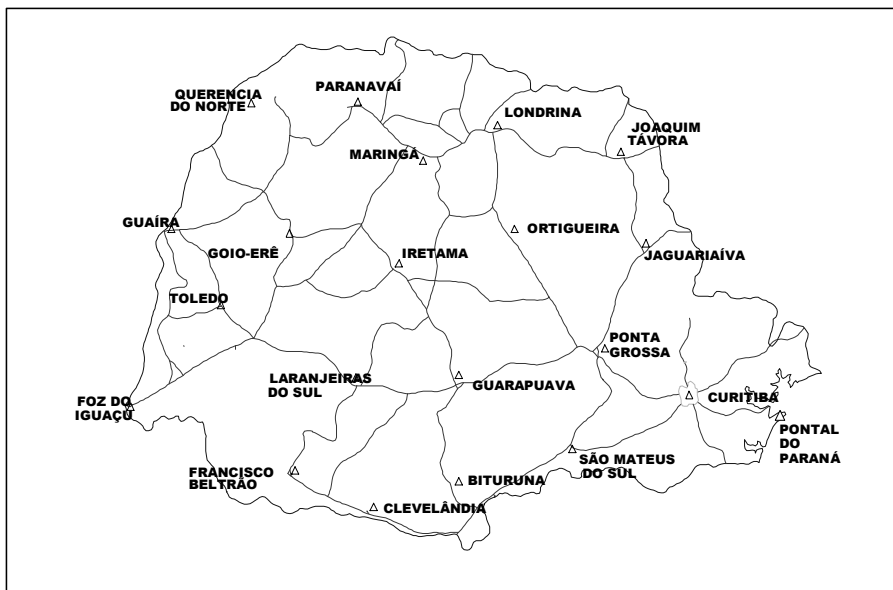


Figure 1. Stations Gravitys.



Figure 2. Reading at the botton of the pilar.



**Figure 3.** Reading at the top of the pilar.

Aiming to impose a scale to the network, the Valinhos station —São Paulo State— was used. This station belongs to RENEGA, we did it also aiming to gauge the gravimeters that were used in the survey.

### **Methodology applied in the network establishment**

We looked for the establishment of a survey methodology, even if was a classic thinking, which could allow the elimination of the principal hindrances derived from the time elimination between the occupation of two known points of control to close the gravity circuits. Micro and macro circuits were formed and the control points injunction were tested via RENEGA.

The survey, in general, had its development according to the following arrangement  $A \rightarrow B \rightarrow A \rightarrow B \rightarrow C \rightarrow B \rightarrow C \rightarrow D \dots \dots J \rightarrow A$ . This arrangement allows the maintenance of the closing control and the determination of two values of the dynamic drift in each interval AB, BC, ..., with the possibility to obtain the variance in each micro circuit, nevertheless saving field displacement, since it is requested one return to the known station (or initial), after occupying sequentially the network stations (Freitas *et al.*, 2002). Actually, eight small circuits were defined (with three

at seven stations) according to Figure 4, aiming the attainment of a more rigorous solution to all the network.

One of these small circuits involved the RENEGA stations Valinhos (São Paulo) and Curitiba (Paraná). These small circuits have their origin from three specific campaigns opening and closing at the RENEGA station in Curitiba. In each station we estimated the use of correction of: the static and dynamic drifts, the luni-solar disturbance and the local dynamic response, by the utilization of local gravity factors already determined in the state of Paraná, via the observation of terrestrial tides (Freitas, 1993). The conception of the macro circuits, smaller circuits with three to five stations, and the closing of micro circuits with multiples lines between two stations, bring up the possibility to test different types of final adjustment of the network, pondering via time performance of the micro circuits or via the determination of the variance of the observations (Freitas *et al.*, 2002).

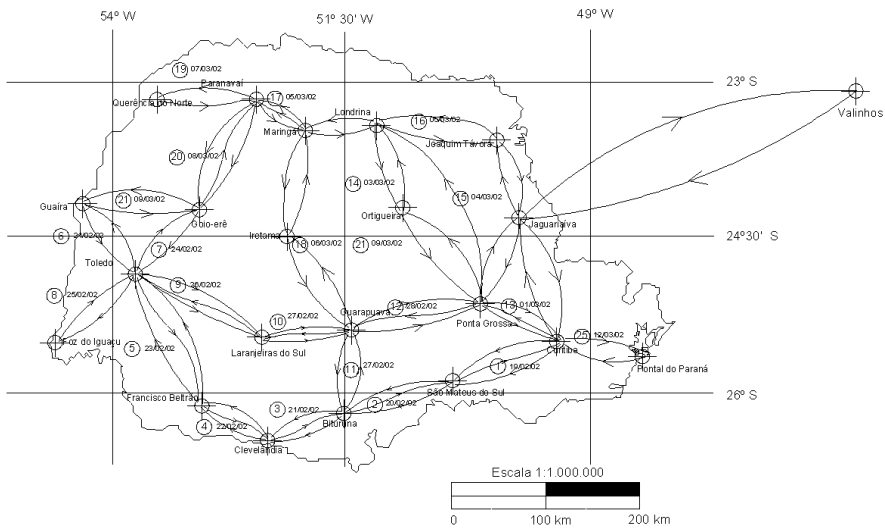


Figure 4. Gravities circuits and the strategy applied.

### Strategy used to network adjustment

Aiming to adjust the gravity network, the least squares method was used, in the parametric form; this strategy makes it possible to calculate quantities indirectly, if these are bound mathematically to other measures which are obtained in a direct form applied.

### ***Individual adjustment***

Gravity differences in levels derived from G-114, G-143, G-372 and Scintrex gravimeters were used in the individual adjustments, taken into consideration in the process of gravity differences in levels the gauge table corrected for the transformation of readings in miligal. To obtain the weight matrix, three different conceptions were pondered. The adjustments represented by the symbols G114, G143, G372 and SCINTREXTEMPO, indicate the equipment applied and that the weight matrix was obtained contrariwise proportional to the time variation employed to close each micro circuit. The adjustments represented by the symbols AGG114, AGG143, AGG372 and SCINTREXVARM, indicate the equipment applied and that the weight matrix was obtained counterclockwise proportional to the variances calculated with two or three observations for each micro circuits, however weighting them according to the number of observations carried out to obtain them. This was done aiming to analyze the amount of solutions from the gravity network regarding the precision, the precision criteria as well the equal eigenvalue test (Santos Junior, 2003; Crossilla & Marchesi, 1983).

### ***Reliability measures***

The reliability theory helps to decide if an error is detectable and what kind of influence the non-detectable error has in the adjustment. So, it is part of a concept to evaluate the quality of the adjustment result. Baard (1968) proposed the application of the global test to detect rough errors and the data snooping test to locate these errors. The errors, which are not eliminated during the observation, give rise to changes in the results, as well in the adjusted parameters. That is why measures to show how much is reliable the observations are needed. One commonly calls these measures “reliability measures”. The reliability concept brought in by Baarda (1968) is subdivided in two: internal reliability and external reliability. The first one, quantifies the smaller portion of the error existing in the observation, which can be found with a certain probability. The second one, quantifies the influence of the non-detectable errors in the adjusted parameters.

### ***Global test***

Aiming to inspect the stochastic model employed, one calculates the statistic:

$$\chi^2_{*2} = \frac{\mathbf{V}^t \mathbf{P} \mathbf{V}}{\sigma_0^2} = \frac{\hat{\sigma}_0^2}{\sigma_0^2} r, \quad (1)$$

which follows the distribution Chi Square to detect the rough errors, where  $r$  is the degree number to maneuver the adjustment. The variance of a unitary weight observation prior  $\hat{\sigma}_0^2$  submitted to a signifying level must be statistically tested with a variance of a unitary weight observation posteriori  $\hat{\sigma}_0^2$ . In the case of the rejection of the calculated statistic, it means that there are indications of rough errors in the observations or in the weight matrix ill guessed or problems over the mathematical model applied, among others.

**Redundancy**

The number of equation superabundant  $r$  in the system of the normal equation is the difference between the number of observation  $n$ , which is equal to the number of equation of observation and the parameters number  $u$ , that are being estimated. The number  $r = n-u$  is called system redundancy. The contribution of each observation to the  $r$  redundancy, is called  $r$  partial redundancy (Förstner, 1979, p. 64) and it is expressed by the relation

$$R = \frac{1}{\hat{\sigma}_0^2} (\sum V)P, \tag{2}$$

where  $\hat{\sigma}_0^2$ , and are, respectively, the variance unity posteriori, the matrix of covariance residual and the weight matrix.

The partial redundancies ( $r_i$ ), calculated from the equation (2), are favorable for the control of the observations. Theses magnitudes show a variation from 0 to 1 (Leick, 1995). According to Kuang (1996), we have two extreme cases to redundancy number  $r_i$ . The first case is the ideal one, where the redundancy number  $r_i = 1$ , although it happens when a measurement is done based on a known quantity; i.e.: a distance measured between two fixed points. In those cases, 100% of all rough errors in the residue  $v_i$  will be exposed and will not have effect over the unknown parameters determination. The second case is where the redundancy number  $r_i = 0$ . In this case, the estimated rough error imbedded in the observation, does not affect at all the residues and then it can not be find out, consequently will be transferred directly to the unknown calculated parameters. If the network is not correctly projected, the individual redundancy number  $r_i$ , can alter significantly and so not remaining close to a average  $r$  (average redundancy number). It means that the controllability is not the same to all the observations. The redundancy number reflects the geometric strength (rigidity) of the geodesic network. In practice, it is desirable to have a network showing a relatively large and uniform redundancy in a

way that one is able to detect rough errors is the same all around it. The Table 1 shows the recommended intervals to guide the decision over the controllability of the observations using partial redundancies.

**Table 1**  
**controllability of the observations using partial redundancies**

<i>Interval</i>	<i>Controlabilidade</i>
$0 \leq r_i < 0,01$	Not have
$0,01 \leq r_i < 0,1$	Bad
$0,1 \leq r_i \leq 0,3$	Sufficient
$0,3 \leq r_i \leq 1$	Good

**Fonte:** Mürle e Bill (1984).

***Internal measures of reliability***

Under a geodesic, network the internal reliability concept has gathered all the criteria which helps detect rough errors (Moraes, 2001).

The internal reliability quantifies the smallest portion of error present during the observation which can be found out with a certain probability, in other words, it indicates the minimum error existing in a test-sensible observation (Förstner, 1979; Benning, 1983; Grimm-Pitzinger and Hanke, 1988, Kuang, 1996).

The minimum error value detectable during the observation is statistically estimated by the equation (Moraes, 2001):

$$\nabla l_{0_i} = \left| \frac{\delta_0}{\sqrt{r_i}} \sigma_{l_i} \right|, (i = 1, \dots, n) \tag{3}$$

where:  $\delta_0$ ,  $\sigma_{l_i}$  e  $r_i$ , e are respectively; the non-centrality parameter, the standard deviation of the i-th observation not adjusted and its respective partial redundancy. The non-centrality parameter ( $\delta_0$ ) is obtained by means of a reduce normal distribution (Kuang, 1996).

The non-centrality parameter means the difference mathematically hopes to get in the null hypothesis and the alternative hypothesis, in other words, the minimum detectable distance between the null hypothesis and the alternative hypothesis. The



$\delta_0$ , values to (to 10 degrees of maneuverability) are charted in Kuang (1996), and are shown in the Table 2.

To see the different degrees of maneuverability of values to  $r$ , please, check the figures in Kavouras (1982) Appendix 3.

**Table 2**  
**Non-centrality parameter in terms of the test power (1-  $\beta_0$ )**  
**and significance level ( $\alpha_0$ )**

Power of Test (1- $\beta_0$ )	Significance of level ( $\alpha_0$ )			
	$\alpha_0 = 0.01\%$	$\alpha_0 = 0.10\%$	$\alpha_0 = 1\%$	$\alpha_0 = 5\%$
50%	3.72	3.29	2.58	1.96
70%	4.41	3.82	3.10	2.48
80%	4.73	4.13	3.42	2.80
90%	5.17	4.57	3.86	3.24
95%	5.54	4.94	4.22	3.61
99%	6.22	5.62	4.90	4.29
99.90%	6.98	6.38	5.67	5.05

**Fonte:** Kuang (1996).

One may see through the equation (3) that  $\nabla l_{0i}$  depends on (Förstner 1979; Benning 1983):

- a) the observation degree of precision which is described by the standard deviation  $\sigma_{ii}$ ;
- b) the network geometry, characterized by the partial redundancy  $r_i$ ;
- c) the signifying level  $\alpha_0$ ;
- d) the quality or the test power;
- e) the non-centrality parameter.

One can see in (3) that the coefficient  $\frac{\delta_0}{\sqrt{r_i}}$  represents the test sensibility. It is better to have small values of  $\frac{\delta_0}{\sqrt{r_i}}$  in a geodesic network. A small  $\frac{\delta_0}{\sqrt{r_i}}$  implies in a high number to  $r_i$ . A big (high) value of  $r_i$ , implies a rough error  $\nabla l_i$  inserted in a observation  $l_i$ , it will be more clearly reflected in the corresponding residue  $v_i$  and consequently it will be easily reveled in the statistic test employed to find out rough errors (data snooping). In other words, the test becomes more sensible and the amount of non-detectable gross errors is reduced to a minimum.

According to Kavouras (1982, p. 75), if the system redundancy is evenly distributed (allocated) in the network, all the partial redundancy numbers and the values obtained to (3) are practically equal. In general, it does not happen. Big (high) differences between the  $r_i$  may be found in different parts of the network, especially in case of different types of observation. For this reason, instead of calculating all the values of the (3) to different values of the ( $r_i$ ), a global internal reliability measure can be employed. An average partial redundancy  $\bar{r}_i$ , is employed, also called relative to all the network given by

$$\bar{r}_i = \frac{\text{tr}(\mathbf{R})}{n} = \frac{r}{n} \tag{4}$$

to calculate (3), obtaining

$$\bar{\nabla}l_{o_i} = \left| \frac{\delta_o}{\sqrt{\bar{r}_i}} \sigma_{l_i} \right| \tag{5}$$

Kavouras (1982), shows through an example that a rough error  $\nabla l_i$  smaller than  $\bar{\nabla}l_{o_i}$  will not be found examining the residues based on the data snooping. The same applies to  $\bar{\nabla}l_{o_i}$ .

***External Reliability Measure***

The external reliability deals with the effect of possible rough errors not detected and localized, in the hidden parameters.

In the adjustment of the observations based on least squares method in the parametric form, the corrections vector  $x$ , which is the solution to normal equations in the adjustment of the geodesic observation, to the presence of a rough error is expressed by (Moraes, 2001):

$$\hat{x} = -\mathbf{N}^{-1} \mathbf{A}^t \mathbf{P} (\mathbf{L} - \mathbf{e}_i \nabla l_i) = -\mathbf{N}^{-1} \mathbf{A}^t \mathbf{P} \mathbf{L} + \mathbf{N}^{-1} \mathbf{A}^t \mathbf{P} \mathbf{e}_i \nabla l_i = -x + \nabla x \tag{6}$$

where

- a)  $\mathbf{N}$  is the matrix of the normal equations coefficient  $\mathbf{A}^t \mathbf{P} \mathbf{A}$  ;
- b)  $\mathbf{A}$  is the matrix of the derivative of observation equations in relation to the hidden ones.

- c) P is the weight matrix of the observations.
- d) L is the difference vector  $l_0 - l_b$  ;
- e) Is the i-th column of an identity matrix  $n \times n$ .

According to Leick (1995, p.168) and Kuang (1996, p. 125), a reckoning to a rough error can be given by:

$$\nabla_i = -\frac{V_i}{r_i} \tag{7}$$

So, the effect of a rough error not detected and calculated by (7) over the solution vector of the adjusted parameters is given by

$$\nabla x = N^{-1}A^T P e_i \nabla_i \tag{8}$$

The equation (8) is sometimes called local external reliability. The minimum error impact that can be detected in the parameters, given by (3), is:

$$\nabla x_{0_i} = N^{-1}A^T P e_i \nabla l_{0_i} \tag{9}$$

Similar to the internal reliability, see equation (5), we can have a global external reliability measure, given by:

$$\bar{\nabla} x = \left( \delta_0 \cdot \sqrt{\frac{u}{r}} \right) \sigma_{u_i} \tag{10}$$

In this case, the effect of the rough error  $\nabla l_i$  not detected in the adjusted parameters can come up to  $\frac{\delta_0 \sqrt{u}}{r}$  times the  $\sigma_{u_i}$  where  $\sigma_{u_i}$  is the precision obtained in the adjusted parameters. Here is desirable to have a small value to the quotient  $\frac{u}{r}$ ,

in other words, the higher the r value the better it is. According to Kavouras (1982), a criteria applied based on practical experience is that the value obtained in (10) is smaller than 10, in other words, it is enough that:

$$\bar{\nabla} x \leq 10. \tag{11}$$

It means that the maximum interval given to the parameters variation can be up to 10 times the precision of the parameters obtained in the adjustments.

**Data Snooping Test**

The data snooping test is frequently employed to analyze the data collected after the adjustment of a geodesic network. Baarda (1968) proposed the data snooping test to localize rough errors, by the examination of the residues got pos-adjustments.

To the no correlated observations, the data snooping test is evaluated by the statistic (Kuang, 1996, p. 132).

$$n_i = \frac{v_i}{\sigma_{v_i}} \sim n(0,1), \tag{12}$$

where  $v_i$  and  $\sigma_{v_i}$  are respectively, the residue and the standard deviation of the residue concerning the  $i$ -th observation.

According to Leick(1995, p. 163), one can represent as follow:

$$\sigma_{v_i} = \hat{\sigma}_0 \sqrt{q_i} = \hat{\sigma}_0 \sqrt{\frac{r_i}{p_i}} = \hat{\sigma}_0 \sqrt{\frac{r_i \sigma_i^2}{\sigma_0^2}} = \frac{\hat{\sigma}_0}{\sigma_0} \sigma_i \sqrt{r_i} \tag{13}$$

It shows that the statistic (12) function the partial redundancy number .

To a certain level of significance , the statistic (12) is compared to a limit value  $k$ . The null hypothesis is rejected if:

$$|n_i| > k, \tag{14}$$

In other words, a rough error is localized in the  $i$ -th observation.

**Reliability Analyzes of the Solutions Obtained from the Individual Adjustments**

To the adjustments called individuals, the global test was employed to a significance level of 5%, the redundancy number to each observation was calculated and the internal and external reliability criteria were employed.

**Global Test**

The global test was employed (Gemael, 1994), where it compared the variance posteriori with the variance prior to a significance level of 5%.

**Table 3**  
**Global Test Results**

<i>Adjustment</i>	$\sigma_0^2$	$\sigma_0^2$	$\chi^{*2}$	$\alpha = 5\%$ $\chi^{*2} < 12.59$ <i>null hypothesis</i>
G114	1	0.000579	0.0035	Accepted
G143	1	0.000158	0.0009	Accepted
G372	1	0.000417	0.0025	Accepted
SCINTREXTEMPO	1	0.000079	0.0005	Accepted
AGG114	1	1.715087	10.29	Accepted
AGG143	1	0.959684	5.75	Accepted
AGG372	1	2.673431	16.04	Rejected
SCINTREXVARM	1	0.111610	0.6697	Accepted
AGG114P	1	3.745743	22.47	Rejected
AGG143P	1	2.560768	15.36	Rejected
AGG372P	1	5.719664	34.32	Rejected
SCINTREXVARMP	1	0.306148	1.84	Accepted

Based on the Table 3 results, we can conclude that the adjustments AGG372, AGG114P, AGG143P and AGG372P did not have a good performance. An indication of this fact could be the presence of rough errors in the observations or weight matrix ill calculated, since the mathematical model applied in the adjustment is quite simple (ordinary).

Another way to make a more carefully evaluation of the adjustments concerning the reliability, is to apply the internal and external reliability criteria.

**Internal Reliability Analysis**

At the beginning, the partial redundancies of all observations were calculated and with these the controllability figures of the observations were checked concerning rough errors.

**Table 4**  
**Observations partial redundancies**

<i>Line</i>	<i>G114</i>	<i>G143</i>	<i>SCINTREX</i>				<i>SCINTREX</i>	
			<i>G372</i>	<i>TEMPO</i>	<i>AGG114</i>	<i>AGG143</i>	<i>AGG372</i>	<i>VARM</i>
01	0.26	0.27	0.27	0.26	0.04	0.64	0.62	0.40
02	0.23	0.23	0.23	0.24	0.37	0.04	0.02	0.17
03	0.23	0.22	0.22	0.22	0.43	0.06	0.13	0.14
04	0.15	0.15	0.15	0.15	0.19	0.03	0.18	0.08
05	0.26	0.27	0.27	0.27	0.12	0.52	0.62	0.34
06	0.31	0.31	0.32	0.32	0.14	0.74	0.10	0.16
07	0.39	0.39	0.39	0.38	0.14	0.34	0.57	0.08
09	0.31	0.31	0.31	0.31	0.13	0.06	0.15	0.24
10	0.18	0.19	0.19	0.19	0.18	0.43	0.56	0.14
11	0.28	0.28	0.28	0.28	0.67	0.17	0.24	0.28
12	0.38	0.38	0.38	0.39	0.01	0.34	0.63	0.19
13	0.22	0.22	0.21	0.22	0.007	0.12	0.16	0.21
14a	0.27	0.28	0.27	0.27	0.78	0.34	0.58	0.34
14b	0.29	0.27	0.27	0.28	0.00006	0.12	0.04	0.26
15a	0.21	0.21	0.20	0.21	0.16	0.39	0.36	0.15
15b	0.22	0.22	0.23	0.21	0.04	0.38	0.02	0.26
16	0.30	0.30	0.30	0.30	0.13	0.02	0.10	0.37
17	0.17	0.17	0.18	0.17	0.52	0.16	0.01	0.12
18a	0.28	0.27	0.27	0.27	0.38	0.39	0.30	0.38
18b	0.26	0.27	0.27	0.28	0.40	0.38	0.10	0.28
19a	0.13	0.13	0.12	0.12	0.39	0.02	0.10	0.10
20	0.24	0.24	0.24	0.25	0.004	0.31	0.11	0.55
21	0.44	0.43	0.43	0.43	0.75	0.004	0.42	0.78

**Table 5**  
**Observations partial redundancies**

<i>Line</i>	<i>AGG114P</i>	<i>AGG143P</i>	<i>AGG372P</i>	<i>SCINTREXVARMP</i>
01	0.03	0.62	0.58	0.39
02	0.31	0.04	0.02	0.16
03	0.42	0.06	0.13	0.13
04	0.19	0.03	0.02	0.08
05	0.12	0.48	0.58	0.31
06	0.14	0.74	0.11	0.16
07	0.14	0.34	0.58	0.08
09	0.11	0.05	0.14	0.22
10	0.16	0.39	0.51	0.13
11	0.75	0.23	0.32	0.36
12	0.01	0.29	0.55	0.16
13	0.01	0.11	0.15	0.20
14a	0.78	0.36	0.59	0.34
14b	0.00006	0.13	0.04	0.26
15a	0.16	0.39	0.38	0.15
15b	0.04	0.38	0.02	0.26
16	0.14	0.02	0.10	0.37
17	0.54	0.18	0.01	0.12
18a	0.38	0.40	0.36	0.38
18b	0.41	0.39	0.12	0.28
19a	0.41	0.03	0.13	0.10
20	0.004	0.34	0.14	0.57
21	0.75	0.004	0.44	0.79

The tables 6 and 7 show the minimum errors calculation ( ) that can be localized in the observations .

**Table 6**  
**Minimum error calculation  $\nabla I_{0_i}$  that can be localized with**  
 $\alpha_0 = 0.45\%, (1-\beta_0) = 80\%$  and so  $\delta_0 = 3.70$

Line	SCINTREX				SCINTREX			
	G114 (mGal)	G143 (mGal)	G372 (mGal)	TEMPO (mGal)	AGG114 (mGal)	AGG143 (mGal)	AGG372 (mGal)	VARM (mGal)
01	21.364	21.233	21.315	21.175	0.240	0.335	0.482	0.518
02	21.364	21.233	21.315	21.175	0.240	0.335	0.482	0.518
03	21.096	21.124	21.193	21.032	0.409	0.257	0.540	0.451
04	21.096	21.124	21.193	21.032	0.409	0.257	0.540	0.451
05	21.096	21.124	21.193	21.032	0.409	0.257	0.540	0.451
06	15.562	15.468	15.441	15.468	0.248	0.177	0.218	0.517
07	13.895	13.835	13.829	13.764	0.220	0.164	0.220	0.418
09	17.741	17.785	17.852	17.731	0.325	0.234	0.400	0.419
10	17.741	17.785	17.852	17.731	0.325	0.234	0.400	0.419
11	16.702	16.695	16.746	16.661	0.253	0.218	0.394	0.373
12	15.770	15.688	15.800	15.819	0.215	0.247	0.238	0.409
13	21.364	21.233	21.315	21.175	0.240	0.335	0.482	0.518
14a	16.002	15.929	16.017	15.992	0.341	0.251	0.145	0.565
14b	16.002	15.929	16.017	15.992	0.341	0.251	0.145	0.565
15a	17.832	17.800	17.841	17.817	0.298	0.306	0.141	0.669
15b	17.832	17.800	17.841	17.817	0.298	0.306	0.141	0.669
16	17.832	17.800	17.841	17.817	0.298	0.306	0.141	0.669
17	17.779	17.758	17.859	17.825	0.437	0.287	0.230	0.573
18a	16.756	16.786	16.839	16.800	0.503	0.332	0.220	0.597
18b	16.756	16.786	16.839	16.800	0.503	0.332	0.220	0.597
19a	19.755	19.754	19.775	19.660	0.424	0.287	0.364	0.673
20	19.755	19.754	19.775	19.660	0.424	0.287	0.364	0.673
21	15.562	15.468	15.441	15.468	0.248	0.177	0.218	0.517



**Table 7**  
**Minimum error calculation that can be localized with**  
 $\alpha_0 = 0.45\%, (1 - \beta_0) = 80\%$  , and so  $\delta_0 = 3.70$

<i>Line</i>	<i>AGG114P</i> (mGal)	<i>AGG143P</i> (mGal)	<i>AGG372P</i> (mGal)	<i>SCINTREXVARMP</i> (mGal)
01	0.152	0.197	0.288	0.306
02	0.152	0.197	0.288	0.306
03	0.239	0.154	0.323	0.270
04	0.239	0.154	0.323	0.270
05	0.239	0.154	0.323	0.270
06	0.175	0.125	0.153	0.365
07	0.155	0.115	0.154	0.293
09	0.201	0.142	0.243	0.255
10	0.201	0.142	0.243	0.255
11	0.169	0.132	0.243	0.230
12	0.139	0.154	0.148	0.256
13	0.152	0.197	0.288	0.306
14a	0.241	0.174	0.102	0.396
14b	0.241	0.174	0.102	0.396
15a	0.210	0.215	0.098	0.471
15b	0.210	0.215	0.098	0.471
16	0.210	0.215	0.098	0.471
17	0.306	0.193	0.146	0.398
18a	0.354	0.232	0.143	0.419
18b	0.354	0.232	0.143	0.419
19a	0.292	0.193	0.229	0.467
20	0.292	0.193	0.229	0.467
21	0.175	0.125	0.153	0.365

Now, with the minimum error calculation that can be localized via the data snooping test, shown at the Tables 6 and 7, it is possible to have a notion of the error magnitude, which is searched in the observations. Applying the data snooping test given by (12) with and so , the way it was expected due to the precision which is guiding the work, no rough error was localized in the observations concerning the twelve solutions analyzed.

**External Reliability Analysis**

As expected, checking the observation residues with the application of the data snooping test, no rough error was noticed in the observations. Then, the goal here is to evaluate the influence of the non-detected errors over calculated parameters.

It is obvious, if the (9) is employed; we will obtain the maximum influence that the parameters can receive from the possible non-detected errors. So, we will employ (7) to calculate the rough error hidden in each observation and consequently the evaluation of the non-detected rough errors influence will be done calculating it via (8). The calculation of the rough error hidden in each observation is shown in the Tables 8 and 9. The influence of these rough errors over the parameters, as well the precision obtained for the parameters, is shown in the Tables 10 to 15.

**Table 8**  
**Rough error calculation ( $\nabla_i$ ) in each observation**

Line				SCINTREX			SCINTREX	
	G114 (mGal)	G143 (mGal)	G372 (mGal)	TEMPO (mGal)	AGG114 (mGal)	AGG143 (mGal)	AGG372 (mGal)	VARM (mGal)
01	0.050	0.074	-0.092	0.073	0.091	0.082	-0.133	0.082
02	0.050	0.074	-0.092	0.073	0.091	0.082	-0.133	0.082
03	-0.094	-0.092	0.167	0.049	0.009	-0.107	0.195	0.052
04	-0.094	-0.092	0.167	0.049	0.009	-0.107	0.195	0.052
05	0.094	0.092	-0.167	-0.049	-0.009	0.107	-0.195	-0.052
06	-0.039	0.045	0.018	-0.016	0.005	0.039	0.019	-0.016
07	0.117	0.002	-0.058	-0.011	0.089	0.014	-0.033	-0.008
09	-0.175	0.120	0.003	-0.037	-0.191	0.129	0.065	-0.047
10	-0.175	0.120	0.003	-0.037	-0.191	0.129	0.065	-0.047
11	-0.089	-0.103	0.161	-0.015	-0.097	-0.112	0.193	-0.007
12	0.114	0.019	-0.038	0.056	0.128	0.013	0.046	0.065
13	0.050	0.074	-0.092	0.073	0.091	0.082	-0.133	0.082
14a	-0.037	0.030	0.081	-0.058	-0.008	0.038	0.108	-0.058
14b	-0.037	0.030	0.081	-0.058	-0.008	0.038	0.108	-0.058
15a	-0.065	-0.010	-0.117	0.054	-0.100	-0.008	-0.129	0.046
15b	0.065	0.010	0.117	-0.054	0.100	0.008	0.129	-0.046
16	-0.065	-0.010	-0.117	0.054	-0.100	-0.008	-0.129	0.046
17	0.110	-0.028	0.016	0.019	0.229	-0.042	0.073	0.026
18a	-0.117	0.025	0.122	0.015	-0.140	0.025	0.118	0.023
18b	0.117	-0.025	-0.122	-0.015	0.140	-0.025	-0.118	-0.023
19a	0.299	-0.068	-0.148	0.003	0.314	-0.060	-0.143	0.007
20	0.299	-0.068	-0.148	0.003	0.314	-0.060	-0.143	0.007
21	0.039	-0.045	-0.018	0.016	-0.005	-0.039	-0.019	0.016

**Table 9**  
**Rough error calculation ( $\nabla_i$ ) in each observation**

<i>Line</i>	<i>AGG114P</i> ( <i>mGal</i> )	<i>AGG143P</i> ( <i>mGal</i> )	<i>AGG372P</i> ( <i>mGal</i> )	<i>SCINTREXVARMP</i> ( <i>mGal</i> )
01	0.092	0.074	-0.117	0.086
02	0.092	0.074	-0.117	0.086
03	0.035	-0.112	0.173	0.057
04	0.035	-0.112	0.173	0.057
05	-0.035	0.112	-0.173	-0.057
06	0.004	0.040	0.023	-0.016
07	0.091	0.012	-0.036	-0.008
09	-0.172	0.129	0.050	-0.053
10	-0.172	0.129	0.050	-0.053
11	-0.095	-0.116	0.181	-0.007
12	0.125	0.017	0.038	0.071
13	0.092	0.074	-0.117	0.086
14a	-0.009	0.040	0.104	-0.058
14b	-0.009	0.040	0.104	-0.058
15a	-0.103	-0.006	-0.127	0.046
15b	0.103	0.006	0.127	-0.046
16	-0.103	-0.006	-0.127	0.046
17	0.233	-0.044	0.066	0.026
18a	-0.146	0.027	0.115	0.023
18b	0.146	-0.027	-0.115	-0.023
19a	0.313	-0.063	-0.132	0.007
20	0.313	-0.063	-0.132	0.007
21	-0.004	-0.040	-0.023	0.016

**Table 10**  
**Influence of the rough errors calculated ( $\nabla_i$ ) over the parameters**

<i>Parameters</i>	<i>G114</i>		<i>G143</i>	
	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)
São Mateus do Sul	0.061	0.018	0.032	-0.002
Bituruna	0.069	0.040	0.036	0.005
Clevelândia	0.085	0.046	0.044	-0.001
Francisco Beltrão	0.089	0.018	0.046	-0.034
Toledo	0.083	0.041	0.043	-0.022
Laranjeiras do sul	0.079	-0.008	0.041	0.015
Guarapuava	0.068	0.032	0.035	-0.010
Ponta Grossa	0.058	-0.023	0.030	-0.012
Jaguariaíva	0.074	-0.018	0.038	-0.008
Joaquim Távora	0.080	-0.016	0.041	-0.004
Ortigueira	0.072	-0.028	0.037	-0.006
Londrina	0.074	-0.036	0.038	-0.002
Maringá	0.076	-0.007	0.040	-0.007
Iretama	0.078	0.016	0.040	-0.009
Paranavaí	0.083	0.128	0.043	-0.039
Goio-erê	0.086	0.112	0.045	-0.036
Guaira	0.092	0.064	0.048	-0.020

**Table 11**  
**Influence of the rough errors calculated ( $\nabla_i$ ) over the parameters**

<i>Parameters</i>	<i>G372</i>		<i>SCINTREXTEMPO</i>	
	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)
São Mateus do Sul	0.052	-0.002	0.022	-0.001
Bituruna	0.059	-0.017	0.025	0.005
Clevelândia	0.072	-0.012	0.031	0.006
Francisco Beltrão	0.075	0.042	0.033	0.021
Toledo	0.070	0.013	0.030	0.010
Laranjeiras do sul	0.067	0.010	0.029	0.000
Guarapuava	0.057	0.007	0.025	0.009
Ponta Grossa	0.048	0.021	0.021	-0.011
Jaguariaíva	0.061	0.046	0.027	-0.017
Joaquim Távora	0.067	0.057	0.029	-0.023
Ortigueira	0.061	0.029	0.026	-0.010
Londrina	0.063	0.037	0.027	-0.011
Maringá	0.065	0.043	0.028	0.001
Iretama	0.066	0.025	0.029	0.005
Paranavaí	0.070	-0.027	0.030	0.004
Goio-erê	0.073	-0.021	0.032	0.008
Guaira	0.078	0.001	0.034	0.007

**Table 12**  
**Influence of the rough errors calculated ( $\nabla_i$ ) over the parameters**

<i>Parameters</i>	<i>AGG114</i>		<i>AGG143</i>	
	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)
São Mateus do Sul	0.016	0.073	0.043	-0.136
Bituruna	0.042	-0.023	0.041	-0.069
Clevelândia	0.073	-0.168	0.044	-0.146
Francisco Beltrão	0.066	-0.226	0.044	-0.238
Toledo	0.054	-0.260	0.046	-0.076
Laranjeiras do sul	0.046	-0.203	0.046	-0.173
Guarapuava	0.011	-0.208	0.039	-0.044
Ponta Grossa	0.007	-0.087	0.029	-0.042
Jaguariaíva	0.039	-0.082	0.048	-0.043
Joaquim Távora	0.043	-0.006	0.044	-0.044
Ortigueira	0.050	0.005	0.042	-0.020
Londrina	0.050	0.013	0.043	-0.037
Maringá	0.074	-0.347	0.046	-0.046
Iretama	0.088	-0.284	0.054	-0.045
Paranavaí	0.059	-0.598	0.046	-0.100
Goio-erê	0.058	-0.289	0.048	-0.084
Guaira	0.062	-0.261	0.048	-0.123

**Table 13**  
**Influence of the rough errors calculated ( $\nabla_i$ ) over the parameters**

<i>Parameters</i>	<i>AGG372</i>		<i>SCINTREXVARM</i>	
	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)
São Mateus do Sul	0.103	0.267	0.023	-0.057
Bituruna	0.102	0.148	0.023	-0.033
Clevelândia	0.125	0.227	0.026	-0.018
Francisco Beltrão	0.127	0.406	0.027	0.012
Toledo	0.109	0.070	0.027	-0.026
Laranjeiras do sul	0.117	0.053	0.025	-0.038
Guarapuava	0.089	0.170	0.022	-0.025
Ponta Grossa	0.079	0.027	0.019	-0.011
Jaguariaíva	0.083	-0.069	0.029	-0.033
Joaquim Távora	0.083	0.046	0.035	-0.037
Ortigueira	0.084	0.204	0.030	-0.026
Londrina	0.083	0.115	0.031	-0.025
Maringá	0.084	0.185	0.031	-0.011
Iretama	0.089	0.115	0.031	-0.022
Paranavaí	0.096	0.110	0.034	-0.008
Goio-erê	0.104	0.040	0.029	-0.019
Guaira	0.110	0.076	0.032	-0.036

**Table 14**  
**Influence of the rough errors calculated ( $\nabla_i$ ) over the parameters**

<i>Parameters</i>	<i>AGG114P</i>		<i>AGG143P</i>	
	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)
São Mateus do Sul	0.013	0.076	0.041	-0.102
Bituruna	0.038	0.001	0.040	-0.040
Clevelândia	0.063	-0.185	0.042	-0.121
Francisco Beltrão	0.057	-0.247	0.043	-0.219
Toledo	0.047	-0.275	0.045	-0.061
Laranjeiras do sul	0.039	-0.215	0.045	-0.160
Guarapuava	0.009	-0.207	0.038	-0.047
Ponta Grossa	0.006	-0.089	0.027	-0.042
Jaguariaíva	0.041	-0.082	0.053	-0.041
Joaquim Távora	0.044	-0.004	0.046	-0.040
Ortigueira	0.052	0.007	0.044	-0.017
Londrina	0.052	0.016	0.045	-0.034
Maringá	0.076	-0.348	0.048	-0.041
Iretama	0.091	-0.284	0.058	-0.045
Paranavaí	0.054	-0.612	0.049	-0.096
Goio-erê	0.053	-0.305	0.048	-0.070
Guaira	0.057	-0.276	0.048	-0.110

**Table 15**  
**Influence of the rough errors calculated ( $\nabla_i$ ) over the parameters**

<i>Parameters</i>	<i>AGG372P</i>		<i>SCINTREXVARMP</i>	
	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)	$\sigma_{u_i}$ (mGal)	<i>Influence of (<math>\nabla_i</math>)</i> (mGal)
São Mateus do Sul	0.092	0.183	0.022	-0.064
Bituruna	0.091	0.077	0.023	-0.040
Clevelândia	0.109	0.163	0.025	-0.024
Francisco Beltrão	0.110	0.324	0.026	0.009
Toledo	0.098	0.095	0.026	-0.033
Laranjeiras do sul	0.103	0.075	0.024	-0.044
Guarapuava	0.078	0.138	0.021	-0.029
Ponta Grossa	0.067	0.037	0.018	-0.009
Jaguariaíva	0.073	-0.061	0.031	-0.031
Joaquim Távora	0.073	0.053	0.039	-0.037
Ortigueira	0.073	0.206	0.033	-0.026
Londrina	0.073	0.120	0.034	-0.026
Maringá	0.073	0.183	0.034	-0.014
Iretama	0.079	0.093	0.034	-0.026
Paranavaí	0.087	0.122	0.037	-0.011
Goio-erê	0.095	0.067	0.028	-0.026
Guaira	0.100	0.103	0.033	-0.042

## Analyses of the Results and Conclusion

The implantation of a scientific gravity network in a determined region with quality higher than the IGSN71 one, is possible if two absolute stations are available in the region, which can be accessed in a gap of time smaller than 12 hours and whose gravity values show differences in the extreme values order of the points to be implanted, making possible to create a scale factor for the instruments at the time of the survey. This way, the survey methodology used, satisfied the need, making possible to reach this condition.

In our strategies for the individual adjustment, we realized that the adjustments with data originated from the digital Scintrex gravimeter showed a better 1°precision. This was noticed in the three methodologies used to the weight matrix formation. It is clear that we are interested in a reliable and precise solution. So, aiming to analyze the reliability of the solutions obtained, we applied the reliability criteria.

Among the twelve solutions analyzed, the solutions AGG372, AGG114P, AGG143P and AGG372P were not approved in the global test. An indication of this rejection in the global test by the four solutions may be ill calculated weight matrix.

Considering the partial redundancies with local and internal reliability criteria, one realizes that in all solutions obtained for the network, even individual, the controllability is not the same for all observations, showing big variations in some cases. According to the criteria exposed in Table 1, letting alone the solutions G114, G143 and G372, all the other solutions show some observations without controllability or show a bad one. It does not mean that the observations are infected with rough errors or that such errors cannot be localized. It is only an indication that if such observations are infected by rough errors, these errors are absorbed by the parameters and one will hardly find them examining the residues. In the calculation of the minimum error reckoning, which can be localized applying the data snooping test, still local and internal reliability and this error must be the smallest one possible. The smaller the error the better the situation of the observation concerning the inspection of rough errors is. We can see in the Table 6 that the worst situation regarding the minimum error, which can be localized through the data snooping test, in the solutions G114, G143, G372 and SCINTREXTEMPO.

It may be an indication that the weight matrix when calculated by the inverse of the time spent to close a circuit is not the best option. We have differences between the partial redundancies in different parts of the network. Consequently, the minimum errors reckoning are different too. Applying the global internal reliability, in other word, employing the equation (5), we see that the global reckoning for the minimum error, which can be localized through the data snooping test, is 7.24 for all solutions. Anyhow, with the magnitude notion of this error calculated in the

internal reliability, we realized that when checking the precision with which one is working, the residues through the data snooping test, an observation would hardly be caught with gross error.

Even taking into consideration all these aspects related to the minimum error that is sensible to the test, the data snooping test was applied to all solutions obtained. As expected, no observation was caught with rough error. No errors were localized, although we know that non-detected errors and non-localized ones affect the parameters calculated by the adjustment; to verify the influence of it, we used the external reliability measure. And since no rough error was localized with the data snooping, we concluded that the biggest (highest) rough error, which can influence a determined observation, is given by the calculated value through the equation (3). So, using equation (9) with the value got from equation (3), it is possible to have the maximum influence of the non-detected and non-localized rough errors to which the parameters may be submitted. With the equation (10) it is possible to have an idea of the global external reliability, which fits to all solution obtained in the network. As a global external reliability measure we have  $6.23 \sigma_{ui}$ , in other words, 6.23 times the standard deviation obtained for each parameter; is the maximum variation that we can have for this respective parameter. This value is in accordance to the value described by the equation (11). One must not forget that this analyzes of internal and external reliability is developed for a certain level of reliability and a certain test power.

According to the previously described analyzes, one has the calculation of the maximum error, which may be infecting a determined observation and consequently the parameters. With equation (7) and based on the residues and in the partial redundancies, it is feasible to calculate a possible rough error which may be infecting a certain observation. The values of this reckoning, for all solution obtained to the gravity network, are exposed in the tables 8 and 9.

Analyzing the results exposed in the table 10 and 11, one realizes, based on the solutions G114, G143, G372 and SCINTREXTEMPO, that the influence undergone by the parameters concerning the possible rough errors calculated through the equation (7) is smaller than the precision given to the respective parameters. This is a positive factor, and we always look for this situation. However, analyzing tables 13 and 15, the solutions SCINTREXVARM and SCINTREXVARMP still show influences of the non-detected errors over the respective parameters in better situation in relation to the calculated parameters of the solutions G114, G143 and G372, consequently show a better situation in relation to the other solutions too.

Considering the criteria to analyze the quality of the geodesic network regarding precision and reliability, one comes to the conclusion that the solutions with data originated from the Scintrex gravimeter, in the construction of the three weigh matrix conceptions, are the best ones to preliminary conceive the gravity network.



Anyway, these preliminary results indicate that it is possible to get a precision better than 30 microgal. To get there, one must search for strategies of data integration and global adjustment.

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